STABILITY ANALYSIS OF NEUTRAL-TYPE WITH TIME –VARYING

DELAY VIA NEW INTEGRAL INEQUALITIES

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ABSTRACT

This work studies the stability of linear systems with a time-varying delay. The main problem considered how to estimate single integral term with time-varying delay information appearing in the derivative of Lyapunov-Krasovskii functional. New integral inequalities are developed in this work for this estimation technique. Compared with the frequently used inequalities based on the combination of Wirtinger-based inequality (or Auxiliary function-based inequality) and reciprocally convex lemma. The proposed condition can provide smaller bounding gap without requiring any extra slack matrix.

ชื่อเรื่อง การวิเคราะห์เสถียรภาพของระบบกับตัวหน่วงที่แปรผันตามเวลาโดยใช้อสมการอินทิกรัลใหม่

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บทคัดย่อ

งานนี้ได้ศึกษาเสถียรภาพของระบบเชิงเส้นกับตัวหน่วงที่แปรผันตามเวลาและได้มุ่งศึกษาการประมาณค่าพจน์ของอินทิกรัลกับตัวหน่วงที่แปรผันตามเวลาที่อยู่ในฟังก์ชันไลพูนอฟ และได้ปรับปรุงอสมการอินทิกรัลใหม่ สำหรับการประมาณค่าอสมการใหม่จะถูกเปรียบเทียบกับอสมการฟังก์ชันเสริม และได้ปรับปรุงใช้บทตั้งทำให้ค่าขอบบนของการประมาณค่าดีขึ้น

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CHAPTER I

Introduction and Preliminaries

1.1 Introduction

Time-varying delays are considered into control loops during implementing of practical control systems through communication networks [1]. The stability, as the basic requirement of control systems, may be destroyed due to the presence of time delays. So, the stability analysis of systems with time-varying delays has been becoming a hot topic in the past few decades [2, 3, 4, 5, 6].

In order to obtain delay-dependent criteria via the LKF method, the following double integral term is usually applied in the LKF [9]:

$$V_r(t) = \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}(s)R\dot{x}(s)dsd\theta \tag{1}$$

where $R > 0$ is the Lyapunov matrix to be determined, h is upper bound of time-varying delay (Note that this paper discusses the time-varying delay with zero low bound, i.e., $0 \leq d(t) \leq h$), and $x(t)$ is the system state. Then its derivative includes the following single integral terms with time-varying delay information:

$$S(t) := -\int_{t-d(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds - \int_{t-h}^{t-d(t)} \dot{x}^T(s)R\dot{x}(s)ds \tag{2}$$

This work develops two relaxed integral inequalities to estimate *S(t)* by considering two integral terms together, instead of the two-step estimation method applied in the existing work. The first (or second) proposed inequality is tighter than the one, obtained via the combination of the Wirtinger-based inequality (or the auxiliary function based inequality) and the reciprocally convex lemma, without requiring any extra slack matrix. we study the problem of neutral system with relaxed conditions and a numerical example to show the effectivc ness of the obtained result.

1.2 Preliminaries and Lemma

Consider the following linear system with a time-varying delay:

$$\dot{x}(t) = Ax(t) + A_d x\big(t - d(t)\big) + C\dot{x}\big(t - d(t)\big), t \geq 0$$
$$x(t) = \phi(t), t\epsilon[-h, 0] \tag{3}$$

where $x(t) \in R^n$ is the system state, A and A_d are the system matrices, the initial condition $\phi(t)$ is a continuously differentiable function, and $d(t)$ is the time-varying delay satisfying

$$0 \leq d(t) \leq h \tag{4}$$

and

$$\mu_1 \leq \dot{d}(t) \leq \mu_2 \tag{5}$$

where h, μ_1, and μ_2 are constant.

This paper aims to derive new delay-dependent stability criteria for analyzing the stability of system (3). In this paper, the key problem to be concerned during the criterion-deriving is how to estimate the following single integral term with time-varying delay information:

$$S(t) = -\int_{t-d(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds - \int_{t-h}^{t-d(t)} \dot{x}^T(s)R\dot{x}(s)ds \tag{6}$$

New inequalities for estimating S(t)

Lemma 1 For a symmetric matrix $R > 0$ and any matrix S_1 satisfying $\begin{bmatrix} R_1 & S_1 \\ * & R_1 \end{bmatrix} \geq 0$ with $R_1 = diag\{R, 3R\}$, the $S(t)$ defined in (6) can be estimated as :

$$S(t) \leq -\frac{1}{h}\zeta_1^T(t)\begin{bmatrix} E_1 \\ E_2 \end{bmatrix}^T \begin{bmatrix} R_1 & S_1 \\ * & R_1 \end{bmatrix}\begin{bmatrix} E_1 \\ E_2 \end{bmatrix}\zeta_1(t) \tag{7}$$

where

$$\zeta_1(t) = \left[x^T(t), x^T\big(t-d(t)\big), x^T(t-h), v_1^T(t), v_2^T(t), \dot{x}^T(t-d(t))\right]^T \tag{8}$$

$$E_1 = \begin{bmatrix} \bar{e}_1 - \bar{e}_2 \\ \bar{e}_1 + \bar{e}_2 - 2\bar{e}_4 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} \bar{e}_2 - \bar{e}_3 \\ \bar{e}_2 + \bar{e}_3 - 2\bar{e}_5 \end{bmatrix}$$

$$\bar{e}_i = \left[0_{n\times(i-1)n}, 1, 0_{n\times(5-i)n}\right],\ i = 1,\ 2, \ldots,\ 5$$

$$v_1(t) = \int_{t-d(t)}^{t} \frac{x(s)}{d(t)} ds$$

$$v_2(t) = \int_{t-h}^{t-d(t)} \frac{x(s)}{h-d(t)} ds$$

Proof: By estimating two parts of $S(t)$ respectively via Writinger- based inequality, combining the obtained terms via the reciprocally convex lemma, and following the same lines as in [9] , inequality (7) can be easily obtained. The details are omitted here.

1.3 Basic Concepts

1.3.1 Types of Matrix

Let $M \in R^{n \times m}$, then we have the following definition.

Definition 1 Matrix M is semi-positive definite if $x^T Mx \geq 0$ for all $x \in R^n, x \neq 0$.

Definition 2 Matrix M is positive definite if $x^T Mx > 0$ for all $x \in R^n, x \neq 0$.

Definition 3 Matrix M is semi-negative definite if $x^T Mx \leq 0$ for all $x \in R^n, x \neq 0$..

Definition 4 Matrix M is negative definite if $x^T Mx < 0$ for all $x \in R^n, x \neq 0$.

1.3.2 Notations

We give some important notations will be used throughout this thesis:

$\boldsymbol{R^+}$ denotes the set of all non-negative real number;

$\boldsymbol{R^n}$ denotes the n-dimensional Euclidean space;

$\boldsymbol{M > 0}$ $(\boldsymbol{M \geq 0})$ denotes the square symmetric, M is positive (semi-) definite matrix;

$\boldsymbol{M < 0}$ $(\boldsymbol{M \leq 0})$ denotes the square symmetric, M is negative (semi-) definite matrix;

$\boldsymbol{M > N}$ $(\boldsymbol{M \geq N})$ denotes the M - N matrix is square symmetric positive (semi-) definite matrix;

$\boldsymbol{M < N}$ $(\boldsymbol{M \leq N})$ denotes the M - N matrix is square symmetric negative (semi-) definite matrix;

$\boldsymbol{R^{n \times m}}$ denotes the space of all $(n \times m)$ real matrices;

$\boldsymbol{A^T}$ denotes the transpose of the vector/matrix A;

$\boldsymbol{A^{-1}}$ denotes the inverse of a non-singular matrix A;

I denotes the identity matrix;

1.3.3 Stability of Ordinary Differential Equation

Consider a dynamical system described by

$$\dot{x}(t) = f\big(t, x(t)\big) \qquad (*)$$

Where $x \epsilon R^n$ and f is a vector having components $f_i(t, x_1, \dots, x_n), i = 1,2, \dots, n.$

We shall assume that the f_i are continuous and satisfy standard condition, such as having continuous first partial derivatives so that the solution of (*) exists and is unique for the given initial conditions. If f_i do not depend explicitly on t, (*) is called autonomous (otherwise, nonautonomous).

If $(t, c) = 0$ for all t, where C is some constant vector, then it follows at once from (*) that if

$$x(t_0) = c \text{ then } x(t) = C \text{ for all } t \geq t_0. \qquad (**)$$

Definition The equilibrium point $x = 0$ of the system (**) is

(i) Stable if, for each $\mathcal{E} > 0$, $\delta = \delta(\mathcal{E}, t_0) > 0$ such that

$\|x(t_0)\| < \delta \rightarrow \|x(t)\| < \mathcal{E}$, $\forall t \geq t_0 \geq 0$,

(ii) Unstable if not stable,

(iii) asymptotically stable if it is stable and there is $c = c(t_0) > 0$ such that

$\|x(t)\| \rightarrow 0 \; as \; t \rightarrow \infty$, for all $\|x(t_0)\| < c$.

Theorem Let $x = 0$ be an equilibrium point and $D \subset R^n$ be a domain containing $x = 0$.

Let $V(x); D \rightarrow R$ be continuously differentiable function, such that

$$V(x) = 0 \text{ and } V(x) > 0 \text{ in } D - \{0\},$$

$$\dot{V}(x) \leq 0 \text{ in } D \, .$$

Then, $x = 0$ is stable. Moreover, if

$$\dot{V}(x) < 0 \text{ in } D - \{0\}$$

Then, $x = 0$ is asymptotically stable.

CHAPTER II

Application to a linear system with time-varying delay

2.1 Main Result

Theorem 1. For given scalars h and $\mu_1 \leq \dot{d}(t) \leq \mu_2$, system (1) is asymptotically stable if one of the following conditions holds

C1: [Derived by (7)] there exist a $3n \times 3n$ matrix $P_1 > 0, n \times n$ matrices $Q > 0, R > 0, Z > 0$, and a $2n \times 2n$ matrix S_1 , S_2 such that the following LMIs hold for $\dot{d}(t) \epsilon \{\mu_1, \mu_2\}$:

$$\begin{bmatrix} R_1 & S_1 \\ * & R_1 \end{bmatrix} \geq 0 \quad , \quad \begin{bmatrix} R_2 & S_2 \\ * & R_2 \end{bmatrix} \geq 0 \tag{9}$$

$$\Psi_1 < 0 \quad , \quad \Psi_2 < 0 \tag{10}$$

Where

$$\Psi_1 = \bar{\Xi}_1 + \bar{\Xi}_1^T - \bar{\Xi}_{2a} + \bar{\Xi}_3 \tag{11}$$

$$\Psi_2 = \bar{\Xi}_1 + \bar{\Xi}_1^T - \bar{\Xi}_{3a} + \bar{\Xi}_3 \tag{12}$$

$$\bar{\Xi}_1 = \mathfrak{E}_1^T P_1 \mathfrak{E}_2$$

$$\bar{\Xi}_{2a} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}^T \begin{bmatrix} R_1 & S_1 \\ * & R_1 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}, \ R_1 = \begin{bmatrix} R & 0 \\ 0 & 3R \end{bmatrix}$$

$$\bar{\Xi}_{3a} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}^T \begin{bmatrix} R_2 & S_2 \\ * & R_2 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}, \ R_2 = \begin{bmatrix} R & 0 \\ 0 & 3R \end{bmatrix}$$

$$\bar{\Xi}_3 = \bar{e}_1^T (Q + Z) \bar{e}_1 - \left(1 - \dot{d}(t)\right) \bar{e}_2^T Q \bar{e}_2 - \bar{e}_3^T Z \bar{e}_3 - \left(1 - \dot{d}(t)\right) \bar{e}_6^T M \bar{e}_6$$
$$+ h^2 \bar{e}_s^T R \bar{e}_s + h^2 \bar{e}_s^T M \bar{e}_s$$

$$\bar{e}_s = [A, \ A_d, 0, \ 0, \ 0, C]$$

$$\bar{e}_i = \left[0_{n \times (i-1)n}, 1, 0_{n \times (5-i)n}\right], \ i = 1, \ 2, \dots, \ 6$$

$$E_i = \begin{bmatrix} \bar{e}_i - \bar{e}_{i+1} \\ \bar{e}_i + \bar{e}_{i+1} - 2\bar{e}_{i+3} \end{bmatrix}, \ i = 1, \ 2$$

$$\mathfrak{E}_1 = [\bar{e}_1^T, \ d(t)\bar{e}_4^T, \ (h - d(t))\bar{e}_5^T]^T$$

$$\mathfrak{E}_2 = \left[\bar{e}_s^T, \ \bar{e}_1^T - \left(1 - \dot{d}(t)\right)\bar{e}_2, \ \left(1 - \dot{d}(t)\right)\bar{e}_2^T - \bar{e}_3^T\right]^T$$

$$\eta_1(t) = \left[x^T(t), \ \int_{t-d(t)}^{t} x^T(s) ds, \ \int_{t-h}^{t-d(t)} x^T(s) ds\right]^T$$

Proof: Construct the following candidate LKF

$$V(t) = \eta_1^T(t)P_1\eta_1(t) + \int_{t-d(t)}^{t} x^T(s)Qx(s)ds + \int_{t-h}^{t} x^T(s)Zx(s)ds + \int_{t-h}^{t} \dot{x}^T(s)M\dot{x}(s)ds$$
$$h\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta + h\int_{t-h}^{t}\int_{s}^{t} \dot{x}^T(u)N\dot{x}(u)duds \quad (13)$$

where

$$V_1(t) = \eta_1^T(t)P_1\eta_1(t) \quad (14)$$

$$V_2(t) = \int_{t-d(t)}^{t} x^T(s)Qx(s)ds + \int_{t-h}^{t} x^T(s)Zx(s)ds + \int_{t-h}^{t} \dot{x}^T(s)M\dot{x}(s)ds$$
$$+h\int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T R\dot{x}(s)dsd\theta + h\int_{t-h}^{t}\int_{s}^{t} \dot{x}^T(u)N\dot{x}(u)duds \quad (15)$$

Calculating the derivative of $V_1(t)$, $V_2(t)$ yield

$$\begin{aligned} V_1(t) &= \eta_1^T(t)P_1\dot{\eta}_1(t) + \dot{\eta}_1^T(t)P_1\eta_1(t) \\ &= 2\eta_1^T(t)P_1\dot{\eta}_1(t) \\ &= 2\left(\left[x^T(t), \int_{t-d(t)}^{t} x^T(s)ds, \int_{t-h}^{t-d(t)} x^T(s)ds\right] P_1 \left[\dot{x}(t), x^T(t) - \left(1-\dot{d}(t)\right)x^T(t-d(t),\right.\right. \\ &\quad \left.\left.(1-\dot{d}(t)x^T\big(t-d(t)\big) - x(t-h)\right]\right) \\ &= 2\left(\left[\bar{e}_1^T, d(t)\bar{e}_4^T, \big(h-d(t)\big)\bar{e}_5^T\right]P_1\left[\bar{e}_s, \bar{e}_1 - \left(1-\dot{d}(t)\right)\bar{e}_2, \left(1-\dot{d}(t)\right)\bar{e}_2 - \bar{e}_3\right]\right) \\ &= \left(\left[\bar{e}_1^T, d(t)\bar{e}_4^T, \big(h-d(t)\big)\bar{e}_5^T\right]P_1\left[\bar{e}_s, \bar{e}_1 - \left(1-\dot{d}(t)\right)\bar{e}_2, \left(1-\dot{d}(t)\right)\bar{e}_2 - \bar{e}_3\right]\right) \\ &\quad + \left(\left[\bar{e}_1, d(t)\bar{e}_4, \big(h-d(t)\big)\bar{e}_5\right]P_1\left[\bar{e}_s^T, \ \bar{e}_1^T - \left(1-\dot{d}(t)\right)\bar{e}_2, \ \left(1-\dot{d}(t)\right)\bar{e}_2^T - \bar{e}_3^T\right]\right) \end{aligned}$$

$$\dot{V}_1(t) = \zeta_1^T(t)(\bar{\Xi}_1 + \bar{\Xi}_1^T)\zeta_1(t) \quad (16)$$

$$\begin{aligned} V_2(t) &= x^T(t)(Q+Z)x(t) - \left(1-\dot{d}(t)\right)x^T\big(t-d(t)\big)Qx\big(t-d(t)\big) - x^T(t-h)Zx(t-h) \\ &\quad + \dot{x}^T(t)M\dot{x}(t) - \left(1-\dot{d}(t)\right)\dot{x}^T\big(t-d(t)\big)M\dot{x}\big(t-d(t)\big) \\ &\quad + h^2\dot{x}^T(t)R\dot{x}(t) - h\int_{t-d(t)}^{t}\dot{x}^T(s)R\dot{x}(s)ds - h\int_{t-h}^{t-d(t)}\dot{x}^T(s)R\dot{x}(s)ds \\ &\quad + h^2\dot{x}^T(t)M\dot{x}(t) - h\int_{t-d(t)}^{t}\dot{x}^T(s)N\dot{x}(s)ds - h\int_{t-h}^{t-d(t)}\dot{x}^T(s)N\dot{x}(s)ds \\ &= \bar{e}_1^T(Q+Z)\bar{e}_1 - \left(1-\dot{d}(t)\right)\bar{e}_2^TQ\bar{e}_2 - \bar{e}_3^TZ\bar{e}_3 - \left(1-\dot{d}(t)\right)\bar{e}_6^TM\bar{e}_6 \\ &\quad + h^2\bar{e}_s^TR\bar{e}_s + h^2\bar{e}_s^TM\bar{e}_s - h\int_{t-d(t)}^{t}\dot{x}^T(s)R\dot{x}(s)ds - h\int_{t-h}^{t-d(t)}\dot{x}^T(s)R\dot{x}(s)ds \\ &\quad - h\int_{t-d(t)}^{t}\dot{x}^T(s)N\dot{x}(s)ds - h\int_{t-h}^{t-d(t)}\dot{x}^T(s)N\dot{x}(s)ds \end{aligned}$$

$$\dot{V}_2 = \zeta_1^T(t)\bar{\Xi}_3\zeta_1(t) - hS_1(t) - hS_2(t) \quad (17)$$

Now, with the help of (16) - (17) , an upper bound of

$$\dot{V}(t) = \zeta_1^T(t)(\bar{\Xi}_1 + \bar{\Xi}_1^T + \bar{\Xi}_3)\zeta_1(t) - hS_1(t) - hS_2(t) \quad . \tag{18}$$

On the one hand by applying Lemma 1, the derivative of $\dot{V}(t)$

$$\dot{V}(t) \leq \zeta_1^T(t)\left\{\bar{\Xi}_1 + \bar{\Xi}_1^{\mathrm{T}} + \bar{\Xi}_3 - \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}^T \left(\begin{bmatrix} R_1 & S_1 \\ * & R_1 \end{bmatrix} + \begin{bmatrix} R_2 & S_2 \\ * & R_2 \end{bmatrix} \right) \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \right\} \zeta_1(t)$$

$$= \zeta_1^T(\Psi_1 + \Psi_2)\zeta_1 \tag{19}$$

CHAPTER III

A Numerical Example

4.1 A Numerical Example

In this section, A numerical example is presented to show the effectiveness improvements of the proposed methods.

Example1. Consider the following linear system (3) with the following parameters ;

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},\ A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},\ C = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix},\ h = 0.01$$

Solution: Theorem1 Solver by Matlab toolbox, we get parameters as following ;

$$P_{11} = \begin{bmatrix} 546.9383 & 0 \\ 0 & 546.9383 \end{bmatrix},\quad P_{12} = (1.0e+03)\begin{bmatrix} -3.3285 & 0 \\ 0 & -3.3285 \end{bmatrix}$$

$$P_{13} = \begin{bmatrix} -57.3453 & 0 \\ 0 & -57.3453 \end{bmatrix},\quad P_{21} = \begin{bmatrix} -21.2758 & 0 \\ 0 & -21.2758 \end{bmatrix}$$

$$P_{22} = \begin{bmatrix} 179.1410 & 0 \\ 0 & 179.1410 \end{bmatrix},\quad P_{23} = \begin{bmatrix} 52.0452 & 0 \\ 0 & 52.0452 \end{bmatrix}$$

$$P_{31} = \begin{bmatrix} -97.4982 & 0 \\ 0 & -97.4982 \end{bmatrix},\quad P_{32} = \begin{bmatrix} 32.7126 & 0 \\ 0 & 32.7126 \end{bmatrix}$$

$$P_{33} = \begin{bmatrix} 32.7126 & 0 \\ 0 & 32.7126 \end{bmatrix},\quad a_{11} = \begin{bmatrix} 28.0044 & 0 \\ 0 & 28.0044 \end{bmatrix}$$

$$Q = \begin{bmatrix} 844.5917 & 0 \\ 0 & 844.5917 \end{bmatrix},\quad Z = \begin{bmatrix} 620.5517 & 0 \\ 0 & 620.5517 \end{bmatrix}$$

$$c_{11} = \begin{bmatrix} 0.1684 & 0 \\ 0 & 0.1684 \end{bmatrix},\quad c_{12} = \begin{bmatrix} -12.1270 & 0 \\ 0 & -12.1270 \end{bmatrix}$$

$$c_{21} = \begin{bmatrix} -9.8050 & 0 \\ 0 & -9.8050 \end{bmatrix},\quad c_{22} = \begin{bmatrix} -15.4035 & 0 \\ 0 & -15.4035 \end{bmatrix}$$

$$g_{11} = \begin{bmatrix} 0.0305 & 0 \\ 0 & 0.0305 \end{bmatrix},\quad g_{12} = \begin{bmatrix} -12.3475 & 0 \\ 0 & -12.3475 \end{bmatrix}$$

$$g_{21} = \begin{bmatrix} -8.5980 & 0 \\ 0 & -8.5980 \end{bmatrix},\quad g_{22} = \begin{bmatrix} -13.3677 & 0 \\ 0 & -13.3677 \end{bmatrix}$$

So, the sufficient condition shows that the trajectory of the solution is stable.

CHAPTER IV

Conclusion

3.1 Conclusion

Theorem 1. For given scalars h and $\mu_1 \leq \dot{d}(t) \leq \mu_2$, system (3) is asymptotically stable if one of the following conditions holds

C1: [Derived by (7)] there exist a $3n \times 3n$ matrix $P_1 > 0, n \times n$ matrices $Q > 0, R > 0, Z > 0$, and a $2n \times 2n$ matrix S_1 , S_2 such that the following LMIs hold for $\dot{d}(t) \epsilon \{\mu_1, \mu_2\}$:

$$\begin{bmatrix} R_1 & S_1 \\ * & R_1 \end{bmatrix} \geq 0 \quad , \quad \begin{bmatrix} R_2 & S_2 \\ * & R_2 \end{bmatrix} \geq 0$$

$$\Psi_1 < 0 \quad , \quad \Psi_2 < 0$$

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APPENDIX

MATLAB CODE

1. MATLAB CODE for finding solution of example

```
[Q+Z+h^2*a11+h^2*M-8*a11+2*P11+2*P21+P12+2*P22 h^2*a11+h^2*M-4*a11-c12-g12-c21-g21-c22-g22+2*P11-P12+2*P13+P23+P31+P32 -c12-g12+c21+g21-c22-g22 -12*a11-P13-P23-P31-P32-P33 2*c12+2*g12+2*c22+2*g22 h^2*a11+h^2*M+2*P11+P13+P21;

h^2*a11+h^2*M-4*a11-2*c12-c21-2*g12-c21-g21-c22-g22-P21-2*P22+P23-P31+P32 -Q+h^2*a11+h^2*M-32*a11-2*c21-2*g21-2*c22-2*g22-P21+2*P22+2*P23+P31+2*P33 -2*a11+c12+g12+c21+g21-c22-g22+P23+P32-P33 12*a11+2*c21+2*g21+2*c22+2*g22 36*a11-2*c12-2*g12+2*c22+2*g22 h^2*a11+h^2*M-P21-P31;

-c12-g12+c21+g21+c22-g22-P13-P23-P31-P32 -12*a11-c12-g12+c21+g21-c22-g22-P13+P23-P31+P32-2*P33 -Z-24*a11+2*P33 -2*c21-2*g21+2*c22+2*g22 36*a11 -P13-P31;

12*a11 12*a11+2*c21+2*g21+2*c22+2*g22 -2*c21-2*g21+2*c22+2*g22 -24*a11 -4*c22-4*g22 0;

2*c12+2*g12+2*c22+2*g22 36*a11+2*c12+2*g12+2*c22+2*g22 36*a11 -4*c22-4*g22 -72*a11 0;

h^2*a11+h^2*M h^2*a11+h^2*M 0 0 0 h^2*a11+h^2*M-M]<0

[a11 0 c11 c12;0 3*a11 c21 c22;c11 c21 a11 0;c12 c22 0 3*a11]>=0

[a11 0 g11 g12;0 3*a11 g21 g22;g11 g21 a11 0;g12 g22 0 3*a11]>=0

[P11 P12 P13;P21 P22 P23;P31 P32 P33]>0
Q>0
Z>0
M>0
```

a_{11}>0

```
setlmis([]);
  P11=lmivar(2,[2,2]);
  P12=lmivar(2,[2,2]);
  P13=lmivar(2,[2,2]);
  P21=lmivar(2,[2,2]);
  P22=lmivar(2,[2,2]);
  P23=lmivar(2,[2,2]);
  P31=lmivar(2,[2,2]);
  P32=lmivar(2,[2,2]);
  P33=lmivar(2,[2,2]);
  Q=lmivar(1,[2,1]);
  a11=lmivar(1,[2,1]);
  Z=lmivar(1,[2,1]);
  c11=lmivar(2,[2,2]);
  c12=lmivar(2,[2,2]);
  c21=lmivar(2,[2,2]);
  c22=lmivar(2,[2,2]);
  g11=lmivar(2,[2,2]);
  g12=lmivar(2,[2,2]);
  g21=lmivar(2,[2,2]);
  g22=lmivar(2,[2,2]);
  M=lmivar(1,[2,1]);
```

```
A=[-2 0;0 -0.9];
Ad=[-1 0;-1 -1];
C=[0 0.2;0.2 0];
h=0.01;

lmiterm([-1 1 1 Q],1,1);                % LMI #1: Q

lmiterm([-2 1 1 a11],1,1);              % LMI #2: a11

lmiterm([-3 1 1 Z],1,1);                % LMI #3: Z

lmiterm([-4 1 1 0],M);                  % LMI #4: M

lmiterm([-5 1 1 a11],1,1);              % LMI #5: a11
lmiterm([-5 2 2 a11],.5*3,1,'s');       % LMI #5: 3*a11 (NON SYMMETRIC?)
lmiterm([-5 3 1 c11],1,1);              % LMI #5: c11
lmiterm([-5 3 2 c21],1,1);              % LMI #5: c21
lmiterm([-5 3 3 a11],1,1);              % LMI #5: a11
lmiterm([-5 4 1 c12],1,1);              % LMI #5: c12
lmiterm([-5 4 2 c22],1,1);              % LMI #5: c22
lmiterm([-5 4 4 a11],.5*3,1,'s');       % LMI #5: 3*a11 (NON SYMMETRIC?)

lmiterm([-6 1 1 a11],1,1);              % LMI #6: a11
lmiterm([-6 2 2 a11],.5*3,1,'s');       % LMI #6: 3*a11 (NON SYMMETRIC?)
lmiterm([-6 3 1 g11],1,1);              % LMI #6: g11
lmiterm([-6 3 2 g21],1,1);              % LMI #6: g21
lmiterm([-6 3 3 a11],1,1);              % LMI #6: a11
lmiterm([-6 4 1 g12],1,1);              % LMI #6: g12
lmiterm([-6 4 2 g22],1,1);              % LMI #6: g22
lmiterm([-6 4 4 a11],.5*3,1,'s');       % LMI #6: 3*a11 (NON SYMMETRIC?)
```

```
lmiterm([-7 1 1 P11],1,1);            % LMI #7: P11
lmiterm([-7 2 1 P21],1,1);            % LMI #7: P21
lmiterm([-7 2 2 P22],1,1);            % LMI #7: P22
lmiterm([-7 3 1 P31],1,1);            % LMI #7: P31
lmiterm([-7 3 2 P32],1,1);            % LMI #7: P32
lmiterm([-7 3 3 P33],1,1);            % LMI #7: P33

lmiterm([8 1 1 Q],1,1);               % LMI #8: Q
lmiterm([8 1 1 Z],1,1);               % LMI #8: Z
lmiterm([8 1 1 a11],.5*h^2,1,'s');    % LMI #8: h^2*a11 (NON SYMMETRIC?)
lmiterm([8 1 1 a11],.5*8,-1,'s');     % LMI #8: -8*a11 (NON SYMMETRIC?)
lmiterm([8 1 1 P11],.5*2,1,'s');      % LMI #8: 2*P11 (NON SYMMETRIC?)
lmiterm([8 1 1 P21],.5*2,1,'s');      % LMI #8: 2*P21 (NON SYMMETRIC?)
lmiterm([8 1 1 P12],1,1);             % LMI #8: P12
lmiterm([8 1 1 P22],.5*2,1,'s');      % LMI #8: 2*P22 (NON SYMMETRIC?)
lmiterm([8 1 1 0],h^2*M);             % LMI #8: h^2*M
lmiterm([8 2 1 a11],h^2,1);           % LMI #8: h^2*a11
lmiterm([8 2 1 a11],4,-1);            % LMI #8: -4*a11
lmiterm([8 2 1 c12],2,-1);            % LMI #8: -2*c12
lmiterm([8 2 1 c21],1,-1);            % LMI #8: -c21
lmiterm([8 2 1 g12],2,-1);            % LMI #8: -2*g12
lmiterm([8 2 1 c21],1,-1);            % LMI #8: -c21
lmiterm([8 2 1 g21],1,-1);            % LMI #8: -g21
lmiterm([8 2 1 c22],1,-1);            % LMI #8: -c22
lmiterm([8 2 1 g22],1,-1);            % LMI #8: -g22
lmiterm([8 2 1 P21],1,-1);            % LMI #8: -P21
lmiterm([8 2 1 P22],2,-1);            % LMI #8: -2*P22
lmiterm([8 2 1 P23],1,1);             % LMI #8: P23
lmiterm([8 2 1 P31],1,-1);            % LMI #8: -P31
lmiterm([8 2 1 P32],1,1);             % LMI #8: P32
lmiterm([8 2 1 0],h^2*M);              % LMI #8: h^2*M
lmiterm([8 2 2 Q],1,-1);              % LMI #8: -Q
```

```
lmiterm([8 2 2 a11],.5*h^2,1,'s');          % LMI #8: h^2*a11 (NON SYMMETRIC?)
lmiterm([8 2 2 a11],.5*32,-1,'s');          % LMI #8: -32*a11 (NON SYMMETRIC?)
lmiterm([8 2 2 c21],.5*2,-1,'s');           % LMI #8: -2*c21 (NON SYMMETRIC?)
lmiterm([8 2 2 g21],.5*2,-1,'s');           % LMI #8: -2*g21 (NON SYMMETRIC?)
lmiterm([8 2 2 c22],.5*2,-1,'s');           % LMI #8: -2*c22 (NON SYMMETRIC?)
lmiterm([8 2 2 g22],.5*2,-1,'s');           % LMI #8: -2*g22 (NON SYMMETRIC?)
lmiterm([8 2 2 P21],1,-1);                  % LMI #8: -P21
lmiterm([8 2 2 P22],.5*2,1,'s');            % LMI #8: 2*P22 (NON SYMMETRIC?)
lmiterm([8 2 2 P23],.5*2,1,'s');            % LMI #8: 2*P23 (NON SYMMETRIC?)
lmiterm([8 2 2 P31],1,1);                   % LMI #8: P31
lmiterm([8 2 2 P33],.5*2,1,'s');            % LMI #8: 2*P33 (NON SYMMETRIC?)
lmiterm([8 2 2 0],h^2*M);                   % LMI #8: h^2*M
lmiterm([8 3 1 c12],1,-1);                  % LMI #8: -c12
lmiterm([8 3 1 g12],1,-1);                  % LMI #8: -g12
lmiterm([8 3 1 c21],1,1);                   % LMI #8: c21
lmiterm([8 3 1 g21],1,1);                   % LMI #8: g21
lmiterm([8 3 1 c22],1,1);                   % LMI #8: c22
lmiterm([8 3 1 g22],1,-1);                  % LMI #8: -g22
lmiterm([8 3 1 P13],1,-1);                  % LMI #8: -P13
lmiterm([8 3 1 P23],1,-1);                  % LMI #8: -P23
lmiterm([8 3 1 P31],1,-1);                  % LMI #8: -P31
lmiterm([8 3 1 P32],1,-1);                  % LMI #8: -P32
lmiterm([8 3 2 a11],12,-1);                 % LMI #8: -12*a11
lmiterm([8 3 2 c12],1,-1);                  % LMI #8: -c12
lmiterm([8 3 2 g12],1,-1);                  % LMI #8: -g12
lmiterm([8 3 2 c21],1,1);                   % LMI #8: c21
lmiterm([8 3 2 g21],1,1);                   % LMI #8: g21
lmiterm([8 3 2 c22],1,-1);                  % LMI #8: -c22
lmiterm([8 3 2 g22],1,-1);                  % LMI #8: -g22
lmiterm([8 3 2 P13],1,-1);                  % LMI #8: -P13
lmiterm([8 3 2 P23],1,1);                   % LMI #8: P23
lmiterm([8 3 2 P31],1,-1);                  % LMI #8: -P31
```

```
lmiterm([8 3 2 P32],1,1);              % LMI #8: P32
lmiterm([8 3 2 P33],2,-1);             % LMI #8: -2*P33
lmiterm([8 3 3 Z],1,-1);               % LMI #8: -Z
lmiterm([8 3 3 a11],.5*24,-1,'s');     % LMI #8: -24*a11 (NON SYMMETRIC?)
lmiterm([8 3 3 P33],.5*2,1,'s');       % LMI #8: 2*P33 (NON SYMMETRIC?)
lmiterm([8 4 1 a11],12,1);             % LMI #8: 12*a11
lmiterm([8 4 2 a11],12,1);             % LMI #8: 12*a11
lmiterm([8 4 2 c21],2,1);              % LMI #8: 2*c21
lmiterm([8 4 2 g21],2,1);              % LMI #8: 2*g21
lmiterm([8 4 2 c22],2,1);              % LMI #8: 2*c22
lmiterm([8 4 2 g22],2,1);              % LMI #8: 2*g22
lmiterm([8 4 3 c21],2,-1);             % LMI #8: -2*c21
lmiterm([8 4 3 g21],2,-1);             % LMI #8: -2*g21
lmiterm([8 4 3 c22],2,1);              % LMI #8: 2*c22
lmiterm([8 4 3 g22],2,1);              % LMI #8: 2*g22
lmiterm([8 4 4 a11],.5*24,-1,'s');     % LMI #8: -24*a11 (NON SYMMETRIC?)
lmiterm([8 5 1 c12],2,1);              % LMI #8: 2*c12
lmiterm([8 5 1 g12],2,1);              % LMI #8: 2*g12
lmiterm([8 5 1 c22],2,1);              % LMI #8: 2*c22
lmiterm([8 5 1 g22],2,1);              % LMI #8: 2*g22
lmiterm([8 5 2 a11],36,1);             % LMI #8: 36*a11
lmiterm([8 5 2 c12],2,1);              % LMI #8: 2*c12
lmiterm([8 5 2 g12],2,1);              % LMI #8: 2*g12
lmiterm([8 5 2 c22],2,1);              % LMI #8: 2*c22
lmiterm([8 5 2 g22],2,1);              % LMI #8: 2*g22
lmiterm([8 5 3 a11],36,1);             % LMI #8: 36*a11
lmiterm([8 5 4 c22],4,-1);             % LMI #8: -4*c22
lmiterm([8 5 4 g22],4,-1);             % LMI #8: -4*g22
lmiterm([8 5 5 a11],.5*72,-1,'s');     % LMI #8: -72*a11 (NON SYMMETRIC?)
lmiterm([8 6 1 a11],h^2,1);            % LMI #8: h^2*a11
lmiterm([8 6 1 0],h^2*M);              % LMI #8: h^2*M
lmiterm([8 6 2 a11],h^2,1);            % LMI #8: h^2*a11
```

```
lmiterm([8 6 2 0],h^2*M);               % LMI #8: h^2*M
lmiterm([8 6 6 a11],.5*h^2,1,'s');      % LMI #8: h^2*a11 (NON SYMMETRIC?)
lmiterm([8 6 6 0],h^2*M-M);             % LMI #8: h^2*M-M

aom=getlmis;

[tmin,xfeas]=feasp(aom)
P11=dec2mat(aom,xfeas,P11)
P12=dec2mat(aom,xfeas,P12)
P13=dec2mat(aom,xfeas,P13)
P21=dec2mat(aom,xfeas,P21)
P22=dec2mat(aom,xfeas,P22)
P23=dec2mat(aom,xfeas,P23)
P31=dec2mat(aom,xfeas,P31)
P32=dec2mat(aom,xfeas,P32)
P33=dec2mat(aom,xfeas,P33)
Q=dec2mat(aom,xfeas,Q)
a11=dec2mat(aom,xfeas,a11)
Z=dec2mat(aom,xfeas,Z)
c11=dec2mat(aom,xfeas,c11)
c12=dec2mat(aom,xfeas,c12)
c21=dec2mat(aom,xfeas,c21)
c22=dec2mat(aom,xfeas,c22)
g11=dec2mat(aom,xfeas,g11)
g12=dec2mat(aom,xfeas,g12)
g21=dec2mat(aom,xfeas,g21)
g22=dec2mat(aom,xfeas,g22)
```

BIOGRAPHY

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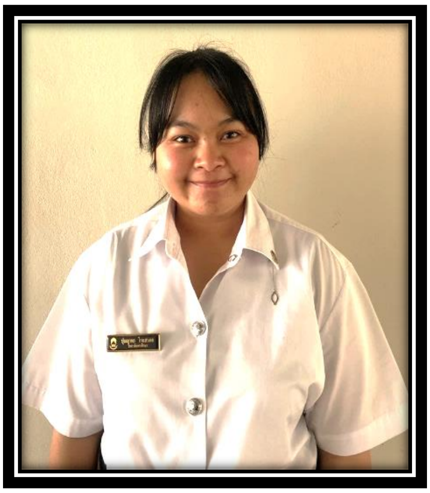

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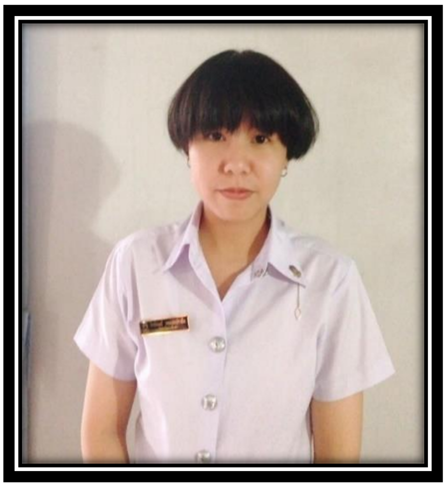
BIOGRAPHY



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