A new result on stability conditions for time-varying delays system with non-linear terms

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An Independent Study Submitted in Partial Fulfillment of the Requirements for the Degree of Bachelor of Science Program in Mathematics

March 2020

University of Phayao

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Advisor and Dean of Science have considered the independent study entitled "A new result on stability conditions for time-varying delays system with non-linear terms"submitted in partial fulfillment of the requirements for Bachelor of Science Degree in Mathematics in hereby approved.

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Darares Phopan

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ชื่อเรื่อง	การศึกษาผลลัพธ์ใหม่เกี่ยวกับความเสถียรภาพของระบบที่มีความล่าช้าที่ขึ้นกับเวลาโดยมีเงื่อนไขที่ไม่เชิงเส้น
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บทคัดย่อ

ในงานวิจัยนี้ได้ศึกษาปัญหาเสถียรภาพของระบบความล่าช้าที่ขึ้นกับเวลาเชิงเส้นแปรผันภายใต้เงื่อนไขไม่เชิงเส้น ให้ความล่าช้าที่ขึ้นกับเวลาโดยการใช้เทคนิคใหม่คือวิธีการจำแนกความล่าช้า ที่ช่วยในการพิจารณาข้อมูลมากขึ้น โดยเงื่อนไขใหม่นี้ได้ใช้วิธีการอสมการแบบอินทิกรัลใหม่ และ ใช้กฎวิธีของนิวตันไลนิช และเงื่อนไขใหม่นี้จะอยู่ในรูปอสมการเมทริกซ์เชิงเส้น โดยได้ใช้ฟังก์ชันไลปูนอฟใหม่ ในการพิจารณาเงื่อนไขให้ดีขึ้น และมีการยกตัวอย่างของตัวเลข เพื่อแสดงให้เห็นถึงประสิทธิภาพของเงื่อนไขใหม่ที่ได้

Title A new result on stability conditions for time-varying delays system with non-linear terms

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Abstract

The issue of stability for linear time-varying delays system under nonlinear terms is discussed, with delays assumed as time-varying. Delay decomposition approach allows information of the delayed plant states to be fully considered. A less conservative delay-dependent robust stability condition is considered, using integral inequality approach to show the relationship of Leibniz–Newton formula terms in the within the framework of linear matrix inequalities (LMIs). Merits of the proposed results lie in lesser conservatism, which are realized by choosing different Lyapunov matrices in the decomposed integral intervals and estimating the upper bound of some cross term more exactly. A numerical example is given to show the effectiveness of the condition.

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CHAPTER 1

Introduction and Preliminaries

1.1 Introduction

Delays are often encountered in various mechanics, physics, biology, medicine, economy, and engineering systems, such as AIDS epidemic, aircraft stabilization, chemical engineering systems, control of epidemics, distributed networks, inferred grinding model manual control, microwave oscillator, models of lasers, neural network, nuclear reactor, population dynamic model, rolling mill, ship stabilization, and systems with lossless transmission lines [1–10]. Stability analysis of dynamic systems with time-delay is thus the focus of theoretical and practical importance, with many works recently paying heed to delay-dependent stability criteria, generally less conservative than delay-independent ones [1,2,4–10]. To derive a less conservative stability criterion, model trans- formation was used in [2] and parameterized neutral model transformation utilized in [14,15].

Motivated by the afore-mentioned analysis, this work deals with delay-dependent stability for a class of time-varying delays system with nonlinear perturbations. By developing delay decomposition approach, information of delayed plant states can be taken into full consideration ,and a numerical example to show the effectiveness of the obtained result.

1.2 Preliminaries

This work considers time-varying delay systems with non-linear perturbations that can be described by linear differential difference equations:

$$\dot{x}(t) - C\dot{x}(t-\tau(t)) = A\,x(t) + Bx(t-h(t)) + Ff(x(t),t) + \mathrm{G}\mathrm{g}(x(t-h(t)),t) \;\; ; \; t > 0 \quad (1)$$

$$x(t+n) = \phi(\eta), \qquad \forall\eta\epsilon[-h,0] \quad (2)$$

with $x(t)\,\epsilon\, R^n$ as state vector of the system, $A, B, C, F, G \in R^{n\times n}$ constant matrices, $\phi(\cdot)$ is continuous vector-valued initial function, $h(t)$ is a time-varying delay in the state, h is an upper bound on delay $h(t)$. $f(x(t),t)\epsilon R^n$, and $g(x(t-h(t)),t)\epsilon R^n$ unknown non-linear perturbations with respect to $x(t)$ and $x(t-h(t))$, respectively, assumed as

$$f^T(x(t),t)f(x(t),t) \leq \alpha^2 x^T(t)x(t) \quad (3)$$

$$\mathrm{g}^{\mathrm{T}}(x(t-h(t)),t)\mathrm{g}(x(t-h(t)),t) \leq \beta^2 x^T(t-h(t))x(t-h(t)) \quad (4)$$

where α and β are known positive constants.

Case I. $h(t)$ is a differentiable function, satisfying for all $t \geq 0$:

$$0 \leq h(t) \leq h \text{ and } \dot{h}(t) \leq h_d \quad (5)$$

$\tau(t)$ is a differentiable function, satisfying for all $t \geq 0$:

$$0 \leq \tau(t) \leq \tau \text{ and } \dot{\tau}(t) \leq \tau_d < 1 \quad (6)$$

1.3 Basic Concepts

1.3.1Types of Matrix

Let $M\epsilon\mathbb{R}^{n\times m}$, then we have the following definition.

Definition 1 Matrix *M* is semi-positive definite if $x^T Mx \geq 0$ for all $x\epsilon R^n, x \neq 0$.

Definition 2 Matrix *M* is positive definite if $x^T Mx > 0$ for all $x\epsilon R^n, x \neq 0$.

Definition 3 Matrix *M* is semi-negative definite if $x^T Mx \leq 0$ for all $x\epsilon R^n, x \neq 0$.

Definition 4 Matrix *M* is negative definite if $x^T Mx < 0$ for all $x\epsilon R^n, x \neq 0$.

1.3.2 Notations

We give some important notations will be used throughout this thesis: R^+denotes the set of all non-negative real number;

R^+ denotes the n-dimensional Euclidean space;

$M > 0$ ($M \geq 0$) denotes the square symmetric, *M* is positive (semi-) definite matrix;

$M < 0$ ($M \leq 0$) denotes the square symmetric, *M* is negative (semi-) definite matrix;

$M > N$ ($M \geq N$) denotes the *M* - *N* matrix is square symmetric positive (semi-) definite matrix;

$M < N$ ($M \leq N$) denotes the *M* - *N* matrix is square symmetric negative (semi-) definite matrix;

$R^{n\times m}$denotes the space of all $(n \times m)$ real matrices;

A^T denotes the transpose of the vector/matrix A;

I denotes the identity matrix;

(A) denotes the set of all eigenvalues of A;

$\lambda max(A) = max\{\mathscr{Re}\,\lambda : \lambda \in \lambda(A)\}$,

$\lambda min(A) = min\{\mathscr{Re}\,\lambda : \lambda \in \lambda(A)\}$;

$\|X\|$ denotes the Euclidean vector norm of x ;

1.3.3 Stability of Ordinary Differential Equation

Consider a dynamical system described by

$$\dot{x}(t) = f\big(t, x(t)\big) \qquad (*)$$

where $x \epsilon R^n$ and f is a vector having components $f_i(t, x_1, \dots, x_n), i = 1,2, \dots, n.$

We shall assume that the f_i are continuous and satisfy standard condition, such as having continuous first partial derivatives so that the solution of (*) exists and is unique for the given initial conditions. If f_i do not depend explicitly on t, (*) is called autonomous (otherwise, nonautonomous).

If $(t, c) = 0$ for all t, where C is some constant vector, then it follows at once from (*) that if $x(t_0) = c$ then $x(t) = C$ for all $t \geq t_0$. Thus solutions starting at C remain there, and C is said to be an equilibrium or critical point. Clearly, by introducing new variables $\acute{x_i} = x_i - c_i$ we can arrange for the equilibrium point to be transferred to the origin; we shall assume that this has been done for any equilibrium point under consideration (there may well be several for a given system (*) so that we then have $(t, 0) = 0, t \geq t_0$).

Definition 5 The equilibrium point $x = 0$ is

(i) Stable if, for each $\mathcal{E} > 0$, there is $\delta = \delta(\mathcal{E}, t_0) > 0$ such that

$$\|x(t_0)\| < \delta \rightarrow \|x(t)\| < \mathcal{E}, \forall t \geq t_0 \geq 0, \qquad (**)$$

(ii) Unstable if not stable,

(iii) asymptotically stable if it is stable and there is $c = c(t_0) > 0$

such that

$$x(t) \to 0 \; as \; t \to \infty, \text{ for all } \|x(t_0)\| < c.$$

Definition 6 [20] A function $V(\cdot) : R^n \to R$ is said to be Lyapunov-Krasovskii functional if it satisfies the following :

1. $V(x)$ and all its partial derivatives $\frac{\partial V}{\partial x_i}$ are continuous.

2. $V(x)$ is positive definite, i.e. $V(0) = 0$ and $V(x) > 0$ for $x \neq 0$ in some neighborhood $\|x\| \leq k$ of the origin.

3. The derivative of V with respect to (2,2), namely

$$\dot{V} = \frac{\partial V}{\partial x_1}\dot{x_1} + \frac{\partial V}{\partial x_2}\dot{x_2} + \cdots + \frac{\partial V}{\partial x_n}\dot{x_n}$$
$$= \frac{\partial V}{\partial x_1}f_1 + \frac{\partial V}{\partial x_2}f_2 + \cdots + \frac{\partial V}{\partial x_n}f_n$$

is negative semi-definite i.e. $\dot{V}(0) = 0$, and for all x satisfy $\|x\| \leq k$, $\dot{V}(0) \leq 0$.

CHAPTER 2

Main Result

Lemma 1. [9,10] For any positive semi-definite matrices

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X^T{}_{12} & X_{22} & X_{23} \\ X^T{}_{13} & X^T{}_{23} & X_{33} \end{bmatrix} \geq 0 \tag{7}$$

the following integral inequality holds

$$-\int_{t-h(t)}^{t} \dot{x}^T(s) X_{33} \dot{x}(s) d \leq \int_{t-h(t)}^{t} [x^T(t) \;\; x^T(t-h(t) \;\; \dot{x}^T(s)\,] \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X^T{}_{12} & X_{22} & X_{23} \\ X^T{}_{13} & X^T{}_{23} & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t) \\ \dot{x}(s) \end{bmatrix} \tag{8}$$

For system (1)–(6) we give stability condition via delay decomposition approach:

Theorem 1. If $0 \leq h(t) \leq \delta h$, for given three scalars $h, \delta,$ and h_d .Then, for any delay $h(t)$ satisfy $0 \leq h(t) \leq h$, $\dot{h}(t) \leq h_d$, and $0 < \delta < 1,$ the system described by (1) with (5) is asymptotically stable if there exist matrices $P = P^{\mathrm{T}} > 0$, $Q_i = Q_i^T > 0$, $R_i = R_i^T > 0$,$(i = 1,2,3,4)$ and positive semi-definite matrices

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X^T{}_{12} & X_{22} & X_{23} \\ X^T{}_{13} & X^T{}_{23} & X_{33} \end{bmatrix} \geq 0$$

$$Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y^T{}_{12} & Y_{22} & Y_{23} \\ Y^T{}_{13} & Y^T{}_{23} & Y_{33} \end{bmatrix} \geq 0$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z^T{}_{12} & Z_{22} & Z_{23} \\ Z^T{}_{13} & Z^T{}_{23} & Z_{33} \end{bmatrix} \geq 0$$

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & 0 & 0 & 0 & \Omega_{18} \\ \Omega_{12}^T & \Omega_{22} & 0 & 0 & \Omega_{25} & 0 & 0 & \Omega_{28} \\ \Omega_{13}^T & 0 & \Omega_{33} & 0 & 0 & 0 & 0 & \Omega_{38} \\ \Omega_{14}^T & 0 & 0 & \Omega_{44} & 0 & 0 & 0 & \Omega_{48} \\ 0 & \Omega_{25}^T & 0 & 0 & \Omega_{55} & \Omega_{56} & 0 & 0 \\ 0 & 0 & 0 & 0 & \Omega_{56}^T & \Omega_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Omega_{77} & 0 \\ \Omega_{18}^T & \Omega_{28}^T & \Omega_{38}^T & \Omega_{48}^T & 0 & 0 & 0 & \Omega_{88} \end{bmatrix} < 0 \tag{9}$$

and $R_1 - X_{33} \geq 0$, and $R_2 - Y_{33} \geq 0$, $R_1 + (1 - h_d)R_3 - Z_{33} \geq 0$ (10)

where

$\Omega_{11} = A^T P + PA + Q_1 + Q_3 + \varepsilon_1 \alpha^2 I + \delta h Z_{11} + Z_{13} + Z_{13}^T$,

$\Omega_{12} = PB + \delta h Z_{12} - Z_{13} + Z_{23}^T$,

$\Omega_{13} = PF$,

$\Omega_{14} = PG$,

$\Omega_{18} = A^T[\delta h R_1 + (1-\delta) h R_2 + \alpha h R_3 + R_4]$,

$\Omega_{22} = -(1 - h_d)Q_3 + \varepsilon_2 \beta^2 I + \delta h X_{11} + X_{13} + X_{13}^T + \delta h Z_{22} - Z_{23} - Z_{23}^T$,

$\Omega_{25} = \delta h X_{12} - X_{13} + X_{23}^T$,

$\Omega_{28} = B^T[\delta h R_1 + (1-\delta) h R_2 + \alpha h R_3 + R_4]$,

$\Omega_{33} = -\varepsilon_1 I$,

$\Omega_{37} = F^T[\delta h R_1 + (1-\delta) h R_2 + \alpha h R_3 + R_4]$,

$\Omega_{44} = -\varepsilon_2 I$,

$\Omega_{47} = G^T[\delta h R_1 + (1-\delta) h R_2 + \alpha h R_3 + R_4]$,

$\Omega_{55} = Q_2 - Q_1 + \delta h X_{22} - X_{23} - X_{23}^T + (1-\delta) h Y_{11} + Y_{13} + Y_{13}^T$,

$\Omega_{56} = (1-\delta) h Y_{12} - Y_{13} + Y_{23}^T$,

$\Omega_{66} = -Q_2 + (1-\delta) h Y_{22} - Y_{23} - Y_{23}^T$,

$\Omega_{77} = -(1 - \tau_d)R_4$,

$\Omega_{88} = -[\delta h R_1 + (1-\delta) h R_2 + \alpha h R_3 + R_4]$.

Proof. A Lyapunov–Krasovskii functional candidate can be constructed as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \quad (11)$$

where

$V_1(t) = x^T(t) P x(t)$

$V_2(t) = \int_{t-\delta h}^{t} x^T(s)\, Q_1 x(s) ds + \int_{t-h}^{t-\delta h} x^T(s) Q_2 x(s) ds + \int_{t-h(t)}^{t} x^T(s) Q_3 x(s) \mathrm{ds}$

$$V_3(t) = \int_{-\delta h}^{0} \int_{t+\theta}^{t} \dot{x}^T(s)\, R_1 \dot{x}(s) ds d\theta + \int_{-h}^{-\delta h} \int_{t+\theta}^{t} \dot{x}^T(s)\, R_2 \dot{x}(s) ds d\theta$$
$$+ \int_{-h(t)}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R_3 \dot{x}(s) ds d\theta + \int_{t-\tau(t)}^{t} \dot{x}^T(s)\, R_4 \dot{x}(s) ds$$

Time derivative of $V(t)$ for $t\epsilon[0,\infty]$ along the trajectory of (1) yield

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) \tag{12}$$

$$\begin{aligned}
\dot{V}_1(t) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \\
&= 2x^T(t)P[Ax(t) + Bx(t-h(t)) + Ff(x(t),t) + Gg(x(t-h(t)),t) \\
&\quad +C\dot{x}(t-\tau(t))] \\
&= x^T(t)(A^TP + PA)x(t) + x^T(t)PBx(t-h(t)) + x^T(t)PFf(x(t),t) \\
&\quad +x^T(t)PGg(x(t-h(t)),t) + x^T(t-h(t))B^TPx(t) \\
&\quad +f^T(x(t),t)F^TPx(t) + g^T(x(t-h(t)),t)G^TPx(t) \\
&\quad +x^T(t)PC\dot{x}(t-\tau(t) + \dot{x}(t-\tau(t)C^TPx(t)
\end{aligned} \tag{13}$$

and

$$\begin{aligned}
\dot{V}_2(t) &= x^T(t)(Q_1 + Q_3)x(t) - x^T(t)(t-h(t))\left(1-\dot{h}(t)\right)Q_3x(t-h(t)) \\
&\quad +x^T(t-\delta h)(Q_2 - Q_1)x(t-\delta h) - x^T(t-h)Q_2x(t-h) \\
&\le x^T(t)(Q_1 + Q_3)x(t) - x^T(t)(t-h(t))(1-h_d)Q_3x(t-h(t)) \\
&\quad +x^T(t-\delta h)(Q_2 - Q_1)x(t-\delta h) - x^T(t-h)Q_2x(t-h)
\end{aligned} \tag{14}$$

$$\begin{aligned}
\dot{V}_3(t) &= \dot{x}^T(t)(\delta hR_1 + (1-\delta)hR_2 + h(t)R_3 + R_4)\dot{x}^T(t) - \int_{t-\delta h}^{t} \dot{x}^T(s)\, R_1\dot{x}(s)ds \\
&\quad - \int_{t-h}^{t-\delta h} \dot{x}^T(s)\, R_2\dot{x}(s)ds - \left(1-\dot{h}(t)\right)\int_{t-h(t)}^{t} \dot{x}^T(s)R_3\dot{x}(s)ds \\
&\quad -(1-(t))(\dot{x}^T(t-\tau(t))R_4\dot{x}(t-\tau(t)) \\
&\le \dot{x}^T(t)(\delta hR_1 + (1-\delta)hR_2 + \alpha hR_3 + R_4)x(t) - \int_{t-\delta h}^{t} \dot{x}^T(s)\, R_1\dot{x}(s)ds \\
&\quad - \int_{t-h}^{t-\delta h} \dot{x}^T(s)\, R_2\dot{x}(s)ds - (1-h_d)\int_{t-h(t)}^{t} \dot{x}^T(s)R_3\dot{x}(s)ds \\
&\quad -\left(1-\dot{\tau}(t)\right)(\dot{x}^T(t-\tau(t))R_4\dot{x}(t-\tau(t))
\end{aligned} \tag{15}$$

Now we estimate the upper bound of the last three terms in inequality (15)

$$\begin{aligned}
&- \int_{t-\delta h}^{t} \dot{x}^T(s)\, R_1\dot{x}(s)ds - \int_{t-h}^{t-\delta h} \dot{x}^T(s)\, R_2\dot{x}(s)ds - (1-h_d)\int_{t-h(t)}^{t} \dot{x}^T(s)R_3\dot{x}(s)ds \\
&= -\int_{t-\delta h}^{t-h(t)} \dot{x}^T(s)\, R_1\dot{x}(s)ds - \int_{t-h}^{t-\delta h} \dot{x}^T(s)\, R_2\dot{x}(s)ds \\
&\quad - \int_{t-h(t)}^{t} \dot{x}^T(s)(R_1 + (1-h_d)R_3)\dot{x}(s)ds \\
&= -\int_{t-\delta h}^{t-h(t)} \dot{x}^T(s)\,(R_1 - X_{33})\dot{x}(s)ds - \int_{t-h}^{t-\delta h} \dot{x}^T(s)\,(R_2 - Y_{33})x^{\cdot}(s)ds
\end{aligned}$$

$$-\int_{t-h(t)}^{t}\dot{x}^T(s)(R_1+(1-h_d)\mathrm{R}_3-Z_{33})\dot{x}(s)ds-\int_{t-\delta h}^{t-h(t)}\dot{x}^T(s)X_{33}\dot{x}(s)ds$$
$$-\int_{t-h}^{t-\delta h}\dot{x}^T(s)\ Y_{33}\dot{x}(s)ds-\int_{t-h(t)}^{t}\dot{x}^T(s)Z_{33}\dot{x}(s)ds \qquad (16)$$

From Lemma 1, we obtain

$$-\int_{t-\delta h}^{t-h(t)}\dot{x}^T(s)X_{33}\dot{x}(s)ds$$
$$\leq \int_{t-\delta h}^{t-h(t)}[x^T(t-h(t))\ \ x^T(t-\delta h)\ \ \dot{x}^T(s)]\begin{bmatrix}X_{11} & X_{12} & X_{13}\\ X^T{}_{12} & X_{22} & X_{23}\\ X^T{}_{13} & X^T{}_{23} & 0\end{bmatrix}\begin{bmatrix}x(t-h(t))\\ x(t-\delta h)\\ \dot{x}(s)\end{bmatrix}$$
$$\leq x^T(t-h(t))(\delta h-h(t))X_{11}x(t-h(t))+x^T(t-h(t))(\delta h-h(t))X_{12}x(t-\delta h)$$
$$+x^T(t-h(t))X_{13}\int_{t-\delta h}^{t-h(t)}\dot{x}(s)\,ds+x^T(t-\delta h)(\delta h-h(t))X_{12}^Tx(t-h(t))$$
$$+x^T(t-\delta h)(\delta h-h(t))X_{22}x(t-\delta h)+x^T(t-\delta h)X_{23}\int_{t-\delta h}^{t-h(t)}\dot{x}(s)\,ds$$
$$+\int_{t-\delta h}^{t-h(t)}\dot{x}^T(s)\,dsX_{13}^Tx(t-h(t))+\int_{t-\delta h}^{t-h(t)}\dot{x}^T(s)\,dsX_{23}^Tx(t-\delta h)$$
$$\leq x^T(t-h(t))X_{11}x(t-h(t))+x^T(t-h(t))\delta hX_{12}x(t-\delta h)$$
$$+x^T(t-h(t))X_{13}\int_{t-\delta h}^{t-h(t)}\dot{x}(s)\,ds+x^T(t-\delta h)\delta hX_{12}^Tx(t-h(t))$$
$$+x^T(t-\delta h)\delta hX_{22}x(t-\delta h)+x^T(t-\delta h)X_{23}\int_{t-\delta h}^{t-h(t)}\dot{x}(s)\,ds$$
$$+\int_{t-\delta h}^{t-h(t)}\dot{x}^T(s)\,dsX_{13}^Tx(t-h(t))+\int_{t-\delta h}^{t-h(t)}\dot{x}^T(s)\,dsX_{23}^Tx(t-\delta h)$$
$$=x^T(t-h(t))[\delta hX_{11}+X_{13}^T+X_{13}]x(t-h(t))$$
$$+x^T(t-h(t))[\delta hX_{12}-X_{13}+X_{23}^T]x(t-\delta h)$$
$$+x^T(t-\delta h)[\delta hX_{12}^T-X_{13}^T+X_{23}]x(t-h(t))$$
$$+x^T(t-\delta h)[\delta hX_{22}-X_{23}-X_{23}^T]x(t-\delta h) \qquad (17)$$

Similarly, we obtain

$$-\int_{t-h}^{t-\delta h}\dot{x}^T(s)\ Y_{33}\dot{x}(s)ds\leq x^T(t-\delta h)[(1-\delta)hY_{11}+Y_{13}^T+Y_{13}]x(t-\delta h)$$
$$+x^T(t-\delta h)[(1-\delta)hY_{12}-Y_{13}+Y_{23}^T]x(t-h)$$
$$+x^T(t-h)[(1-\delta)hY_{12}^T-Y_{13}^T+Y_{23}]x(t-\delta h)$$
$$+x^T(t-h)[(1-\delta)hY_{22}-Y_{23}-Y_{23}^T]x(t-h) \qquad (18)$$

and

$$-\int_{t-h(t)}^{t}\dot{x}^T(s)Z_{33}\dot{x}(s)ds \le x^T(t)[h(t)Z_{11}+Z_{13}^T+Z_{13}]x(t)$$

$$+x^T(t)[h(t)Z_{12}-Z_{13}+Z_{23}^T]x(t-h(t))$$

$$+x^T(t-h(t))[h(t)Z_{12}^T-Z_{13}^T+Z_{23}]x(t)$$

$$+x^T(t-h(t))[h(t)Z_{22}-Z_{23}-Z_{23}^T]x(t-h(t))$$

$$\le x^T(t)[\delta hZ_{11}+Z_{13}^T+Z_{13}]x(t)$$

$$+x^T(t)[\delta hZ_{12}-Z_{13}+Z_{23}^T]x(t-h(t))$$

$$+x^T(t-h(t))[\delta hZ_{12}^T-Z_{13}^T+Z_{23}]x(t)$$

$$+x^T(t-h(t))[\delta hZ_{22}-Z_{23}-Z_{23}^T]x(t-h(t)) \tag{19}$$

The operator for term $\dot{x}^T(t)[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]\,\dot{x}(t)$ is as follows:

$$\dot{x}^T(t)[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]\dot{x}(t)$$

$$=[A\,x(t)+Bx(t-h(t))+Ff(x(t),t)+\mathrm{Gg}(x(t-h(t)),t)\ +C\dot{x}(t-\tau(t))]^T$$

$$\times[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4][\,A\,x(t)+Bx(t-h(t))+Ff(x(t),t)$$

$$+\mathrm{Gg}(x(t-h(t)),t)\ +C\dot{x}(t-\tau(t))]$$

$$=x^T(t)A^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]Ax(t)$$

$$+x^T(t)A^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]Bx(t-h(t))$$

$$+x^T(t)A^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]Ff(x(t),t)$$

$$+x^T(t)A^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]\mathrm{Gg}(x(t-h(t)),t)$$

$$+x^T(t)A^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]C\dot{x}(t-\tau(t))$$

$$+x^T(t-h(t))B^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]Ax(t)$$

$$+x^T(t-h(t))B^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]Bx(t-h(t))$$

$$+x^T(t-h(t))B^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]Ff(x(t),t)$$

$$+x^T(t-h(t))B^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]\ \mathrm{Gg}(x(t-h(t)),t)$$

$$+x^T(t-h(t))B^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]\ C\dot{x}(t-\tau(t))$$

$$+f^T(x(t),t)F^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]\ Ax(t)$$

$$+f^T(x(t),t)F^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]Bx(t-h(t))$$

$$+f^T(x(t),t)F^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]Ff(x(t),t)$$

$$+f^T(x(t),t)F^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]\mathrm{Gg}(x(t-h(t)),t)$$

$$+f^T(x(t),t)F^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]C\dot{x}(t-\tau(t))$$

$$+g^T\big(x\big(t-h(t)\big),t\big)G^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]Ax(t)$$

$$+g^T\big(x\big(t-h(t)\big),t\big)G^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]\,Bx\big(t-h(t)\big)$$

$$+g^T\big(x\big(t-h(t)\big),t\big)G^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]Ff(x(t),t)$$

$$+g^T\big(x\big(t-h(t)\big),t\big)G^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]\mathrm{Gg}(x(t-h(t)),t)$$

$$+g^T(x(t-h(t)),t)G^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]C\dot{x}(t-\tau(t))$$

$$+\dot{x}^T\big(t-\tau(t)\big)C^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]Ax(t)$$

$$+\dot{x}^T\big(t-\tau(t)\big)C^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]Bx\big(t-h(t)\big)$$

$$+\dot{x}^T\big(t-\tau(t)\big)C^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]Ff(x(t),t)$$

$$+\dot{x}^T\big(t-\tau(t)\big)C^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]G\mathrm{g}(x(t-h(t)),t)$$

$$+\dot{x}^T\big(t-\tau(t)\big)C^T[\delta hR_1+(1-\delta)hR_2+\alpha hR_3+R_4]C\dot{x}(t-\tau(t)) \qquad (20)$$

Note that for any $\varepsilon_1 \geq 0$, $\varepsilon_2 \geq 0$, it follows from (3) to (4) that

$$\varepsilon_1[\alpha^2 x^T(t)x(t)-f^T(x(t),t)f(x(t),t)]\geq 0 \qquad (21)$$

and

$$\varepsilon_2[\beta^2 x^T\big(t-h(t)\big)x(t-h(t))-g^T(x(t-h(t)),t)\mathrm{g}(x(t-h(t)),t)]\geq 0 \qquad (22)$$

Combining (12) and (22) yields

$$\dot{V}(t)\;\leq \xi^T(t)\Xi\xi(t)-\int_{t-\alpha h}^{t-h(t)}\dot{x}^T\,(s)\,(R_1-X_{33})\dot{x}(s)ds-\int_{t-h}^{t-\alpha h}\dot{x}^T\,(s)(R_2-Y_{33})\dot{x}(s)ds$$
$$-\int_{t-h(t)}^{t}\dot{x}^T\,(s)(R_1+(1-h_d)R_3-Z_{33})\dot{x}(s)ds \qquad (23)$$

where

$$\xi^T(t)=[x^T(t)\quad x^T\big(t-h(t)\big)\quad f^T(x(t),t)\quad \mathrm{g}^T\big(x\big(t-h(t)\big),t\big)x(t-\delta h)x(t-h)\quad \dot{x}^T\big(t-\tau(t)\big)]$$

$$\Xi=\begin{bmatrix}\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & 0 & 0\\ \Xi_{12}^T & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & 0\\ \Xi_{13}^T & \Xi_{23}^T & \Xi_{33} & \Xi_{34} & 0 & 0\\ \Xi_{14}^T & \Xi_{24}^T & \Xi_{34}^T & \Xi_{44} & 0 & 0\\ 0 & \Xi_{25}^T & 0 & 0 & \Xi_{55} & \Xi_{56}\\ 0 & 0 & 0 & 0 & \Xi_{56}^T & \Xi_{66}\end{bmatrix}<0$$

with

$\Xi_{11} = A^T P + PA + Q_1 + Q_3 + \varepsilon_1 \alpha^2 I + \delta h Z_{11} + Z_{13} + Z_{13}^T \ + A^T[\delta h R_1 + (1-\delta)hR_2 + \alpha h R_3 + R_4]A,$

$\Xi_{12} = PB + \delta h Z_{12} - Z_{13} + Z_{23}^T + A^T[\delta h R_1 + (1-\delta)hR_2 + \alpha h R_3 + R_4]B\ ,$

$\Xi_{13} = PF + A^T[\delta h R_1 + (1-\delta)hR_2 + \alpha h R_3 + R_4]F,$

$\Xi_{14} = PG + A^T[\delta h R_1 + (1-\delta)hR_2 + \alpha h R_3 + R_4]G,$

$\Xi_{22} = -(1-h_d)Q_3 + \varepsilon_2 \beta^2 I + \delta h X_{11} + X_{13} + X_{13}^T + \delta h Z_{22} - Z_{23} - Z_{23}^T$
$\quad + B^T[\delta h R_1 + (1-\delta)hR_2 + \alpha h R_3 + R_4]B,$

$\Xi_{23} = B^T[\delta h R_1 + (1-\delta)hR_2 + \alpha h R_3 + R_4]\ F,$

$\Xi_{24} = B^T[\delta h R_1 + (1-\delta)hR_2 + \alpha h R_3 + R_4]G,$

$\Xi_{25} = \delta h X_{12} - X_{13} + X_{23}^T = \varepsilon_1 I + F^T[\delta h R_1 + (1-\delta)hR_2 + \alpha h R_3 + R_4]F\ ,$

$\Xi_{34} = F^T[\delta h R_1 + (1-\delta)hR_2 + \alpha h R_3 + R_4]G,$

$\Xi_{44} = -\varepsilon_2 I + G^T[\delta h R_1 + (1-\delta)hR_2 + \alpha h R_3 + R_4]G\ ,$

$\Xi_{55} = Q_2 - Q_1 + \delta h X_{22} - X_{23} - X_{23}^T + (1-\delta)hY_{11} + Y_{13} + Y_{13}^T,$

$\Xi_{56} = (1-\delta)hY_{12} - Y_{13} + Y_{23}^T\ ,$

$\Xi_{66} = -Q_2 + (1-\delta)hY_{22} - Y_{23} - Y_{23}^T\ .$

From Eq. (1) and Schur complement, it is easy to see that

$\dot{V}(t) < 0$ holds if $R_1 - X_{33} \geq 0$, $R_2 - Y_{33} \geq 0$, $R_1 + (1-h_d)R_3 - Z_{33} \geq 0$, and

$0 \leq h(t) \leq \delta h.$

CHAPTER 3

A numerical example

3.1 A numerical example

In this section, a numerical example are given to show the effectiveness of the proposed method.

Example 3.1 Consider the system (1) with

$$\dot{x}(t) - C\dot{x}(t-\tau(t)) = A\,x(t) + Bx(t-h(t)) + Ff(x(t),t) + \mathrm{Gg}(x(t-h(t)),t) \tag{1}$$

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix},\quad B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},\quad F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\quad G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\quad C = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix},$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\quad \alpha = 0.05,\quad \beta = 0.1,\quad \tau = 0.5,\quad \delta = 0.2,\quad h = 0.5$$

Solution: Using MATLAB LMI toolbox. Theorem 1 are feasible with solution given by,

$$P = \begin{bmatrix} 44.1689 & -0.3068 \\ -0.3068 & 21.9952 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 41.5944 & -43.3633 \\ -43.3633 & -8.6419 \end{bmatrix},\qquad Q_2 = \begin{bmatrix} 22.7029 & -27.9703 \\ -27.9703 & -9.6759 \end{bmatrix}$$

$$Q_3 = 1 \times 10^8 \begin{bmatrix} 4.9478 & -0.3000 \\ -0.3000 & 5.6606 \end{bmatrix}$$

$$R_1 = \begin{bmatrix} 110.8254 & 0.0287 \\ 0.0287 & 110.8728 \end{bmatrix},\qquad R_2 = \begin{bmatrix} 12.1123 & -0.0101 \\ -0.0101 & 12.1037 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 396.0537 & 0.0463 \\ 0.0463 & 396.0252 \end{bmatrix}.$$

CHAPTER 4

Conclusion

Theorem 1. If $0 \leq h(t) \leq \delta h,$ for given three scalars $h, \delta,$ and h_d .Then, for any delay $h(t)$ satisfy $0 \leq h(t) \leq h, \dot{h}(t) \leq h_d$, and $0 < \delta < 1,$ the system described by (1) with (5) is asymptotically stable if there exist matrices $P = P^{\mathrm{T}} > 0,\ Q_i = Q_i^T > 0, R_i = R_i^T > 0$,$(i = 1,2,3,4)$ and positive semi-definite matrices

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X^T{}_{12} & X_{22} & X_{23} \\ X^T{}_{13} & X^T{}_{23} & X_{33} \end{bmatrix} \geq 0$$

$$Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y^T{}_{12} & Y_{22} & Y_{23} \\ Y^T{}_{13} & Y^T{}_{23} & Y_{33} \end{bmatrix} \geq 0$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z^T{}_{12} & Z_{22} & Z_{23} \\ Z^T{}_{13} & Z^T{}_{23} & Z_{33} \end{bmatrix} \geq 0$$

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & 0 & 0 & 0 & \Omega_{18} \\ \Omega_{12}^T & \Omega_{22} & 0 & 0 & \Omega_{25} & 0 & 0 & \Omega_{28} \\ \Omega_{13}^T & 0 & \Omega_{33} & 0 & 0 & 0 & 0 & \Omega_{38} \\ \Omega_{14}^T & 0 & 0 & \Omega_{44} & 0 & 0 & 0 & \Omega_{48} \\ 0 & \Omega_{25}^T & 0 & 0 & \Omega_{55} & \Omega_{56} & 0 & 0 \\ 0 & 0 & 0 & 0 & \Omega_{56}^T & \Omega_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Omega_{77} & 0 \\ \Omega_{18}^T & \Omega_{28}^T & \Omega_{38}^T & \Omega_{48}^T & 0 & 0 & 0 & \Omega_{88} \end{bmatrix} < 0 \qquad (9)$$

and $R_1 - X_{33} \geq 0$, and $R_2 - Y_{33} \geq 0,\ R_1 + (1 - h_d)R_3 - Z_{33} \geq 0$ (10)

where

$\Omega_{11} = A^T P + PA + Q_1 + Q_3 + \varepsilon_1 \alpha^2 I + \delta h Z_{11} + Z_{13} + Z_{13}^T$,

$\Omega_{12} = PB + \delta h Z_{12} - Z_{13} + Z_{23}^T$,

$\Omega_{13} = PF,$

$\Omega_{14} = PG,$

$\Omega_{18} = A^T[\delta h R_1 + (1 - \delta) h R_2 + \alpha h R_3 + R_4]$,

$\Omega_{22} = -(1 - h_d) Q_3 + \varepsilon_2 \beta^2 I + \delta h X_{11} + X_{13} + X_{13}^T + \delta h Z_{22} - Z_{23} - Z_{23}^T,$

$\Omega_{25} = \delta h X_{12} - X_{13} + X_{23}^T$,

$\Omega_{28} = B^T[\delta h R_1 + (1 - \delta) h R_2 + \alpha h R_3 + R_4]$,

$\Omega_{33} = -\varepsilon_1 I$,

$\Omega_{37} = F^T[\delta h R_1 + (1-\delta)hR_2 + \alpha h R_3 + R_4]$,

$\Omega_{44} = -\varepsilon_2 I$,

$\Omega_{47} = G^T[\delta h R_1 + (1-\delta)hR_2 + \alpha h R_3 + R_4]$,

$\Omega_{55} = Q_2 - Q_1 + \delta h X_{22} - X_{23} - X_{23}^T + (1-\delta)hY_{11} + Y_{13} + Y_{13}^T$,

$\Omega_{56} = (1-\delta)hY_{12} - Y_{13} + Y_{23}^T$,

$\Omega_{66} = -Q_2 + (1-\delta)hY_{22} - Y_{23} - Y_{23}^T$,

$\Omega_{77} = -(1-\tau_d)R_4$,

$\Omega_{88} = -[\delta h R_1 + (1-\delta)hR_2 + \alpha h R_3 + R_4]$.

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BIBLIOGRAPHY

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APPENDIX

MATLAB CODE

```
A=[-2,0;0,-1];
B=[-1,0;-1,-1];
C=[0.05,0;0,0.05];
F=[1,0;0,1];
G=[1,0;0,1];
a=0.05;
b=0.1;
h=0.5;
d=0.2;
I=[1,0;0,1];
t=0.5;

     setlmis([]);
  P=lmivar(1,[2,1]);
  Q1=lmivar(1,[2,1]);
  Q2=lmivar(1,[2,1]);
  Q3=lmivar(1,[2,1]);
  Q4=lmivar(1,[2,1]);
  R1=lmivar(1,[2,1]);
  R2=lmivar(1,[2,1]);
  R3=lmivar(1,[2,1]);
  R4=lmivar(1,[2,1]);
  e1=lmivar(1,[1,1]);
  e2=lmivar(1,[1,1]);
  e3=lmivar(1,[1,1]);

  X11=lmivar(2,[2,2]);
  X12=lmivar(2,[2,2]);
  X13=lmivar(2,[2,2]);
  X22=lmivar(2,[2,2]);
  X23=lmivar(2,[2,2]);
  X33=lmivar(2,[2,2]);
  Y11=lmivar(2,[2,2]);
  Y12=lmivar(2,[2,2]);
  Y13=lmivar(2,[2,2]);
  Y22=lmivar(2,[2,2]);
  Y23=lmivar(2,[2,2]);
  Y33=lmivar(2,[2,2]);
  Z11=lmivar(2,[2,2]);
  Z12=lmivar(2,[2,2]);
  Z13=lmivar(2,[2,2]);
  Z22=lmivar(2,[2,2]);
  Z23=lmivar(2,[2,2]);
  Z33=lmivar(2,[2,2]);


  lmiterm([-1 1 1 P],1,1);                       % LMI #1: P

  lmiterm([-2 1 1 Q1],1,1);                      % LMI #2: Q1

  lmiterm([-3 1 1 Q2],1,1);                      % LMI #3: Q2

  lmiterm([-4 1 1 Q3],1,1);                      % LMI #4: Q3

  lmiterm([-5 1 1 0],Q4);                        % LMI #5: Q4
```

```
  lmiterm([-6 1 1 R1],1,1);                       % LMI #6: R1

  lmiterm([-7 1 1 R2],1,1);                       % LMI #7: R2

  lmiterm([-8 1 1 R3],1,1);                       % LMI #8: R3

  lmiterm([-9 1 1 0],R4);                         % LMI #9: R4

  lmiterm([-10 1 1 0],X11);                       % LMI #10: X11
  lmiterm([-10 2 1 0],X12');                      % LMI #10: X12'
  lmiterm([-10 2 2 0],X22);                       % LMI #10: X22
  lmiterm([-10 3 1 0],X13');                      % LMI #10: X13'
  lmiterm([-10 3 2 0],X23');                      % LMI #10: X23'
  lmiterm([-10 3 3 0],X33);                       % LMI #10: X33

  lmiterm([-11 1 1 0],Y11);                       % LMI #11: Y11
  lmiterm([-11 2 1 0],Y12');                      % LMI #11: Y12'
  lmiterm([-11 2 2 0],Y22);                       % LMI #11: Y22
  lmiterm([-11 3 1 0],Y13');                      % LMI #11: Y13'
  lmiterm([-11 3 2 0],Y23');                      % LMI #11: Y23'
  lmiterm([-11 3 3 0],Y33);                       % LMI #11: Y33

  lmiterm([-12 1 1 0],Z11);                       % LMI #12: Z11
  lmiterm([-12 2 1 0],Z12');                      % LMI #12: Z12'
  lmiterm([-12 2 2 0],Z22);                       % LMI #12: Z22
  lmiterm([-12 3 1 0],Z13');                      % LMI #12: Z13'
  lmiterm([-12 3 2 0],Z23');                      % LMI #12: Z23'
  lmiterm([-12 3 3 0],Z33);                       % LMI #12: Z33

  lmiterm([-13 1 1 R1],1,1);                      % LMI #13: R1
  lmiterm([-13 1 1 0],-X33);                      % LMI #13: -X33

  lmiterm([-14 1 1 R2],1,1);                      % LMI #14: R2
  lmiterm([-14 1 1 0],-Y33);                      % LMI #14: -Y33

  lmiterm([-15 1 1 R1],1,1);                      % LMI #15: R1
  lmiterm([-15 1 1 R3],1,1);                      % LMI #15: R3
  lmiterm([-15 1 1 R3],.5*h*d,-1,'s');            % LMI #15: -h*d*R3 (NON
SYMMETRIC?)
  lmiterm([-15 1 1 0],-Z33);                      % LMI #15: -Z33

  lmiterm([16 1 1 P],A',1,'s');                   % LMI #16: A'*P+P*A
  lmiterm([16 1 1 Q1],1,1);                       % LMI #16: Q1
  lmiterm([16 1 1 0],e1*a^2*I+d*h*Z11+Z13+Z13');     % LMI #16:
e1*a^2*I+d*h*Z11+Z13+Z13'
  lmiterm([16 2 1 -P],1,B');                      % LMI #16: P'*B'
  lmiterm([16 2 1 0],d'*h'*Z12'-Z13'+Z23);        % LMI #16: d'*h'*Z12'-
Z13'+Z23
  lmiterm([16 2 2 Q3],1,-1);                      % LMI #16: -Q3
  lmiterm([16 2 2 Q3],.5*h*d,1,'s');              % LMI #16: h*d*Q3 (NON
SYMMETRIC?)
  lmiterm([16 2 2 0],e3*b^2*I+d*h*X11+X13+d*h*Z22-Z23-Z23');     % LMI #16:
e3*b^2*I+d*h*X11+X13+d*h*Z22-Z23-Z23'
  lmiterm([16 3 1 -P],1,F');                      % LMI #16: P'*F'
  lmiterm([16 3 3 0],-e1*I);                      % LMI #16: -e1*I
  lmiterm([16 4 1 -P],1,G');                      % LMI #16: P'*G'
  lmiterm([16 4 4 0],-e1*I);                      % LMI #16: -e1*I
  lmiterm([16 5 2 0],d'*h'*X12'-X13'+X23);        % LMI #16: d'*h'*X12'-
X13'+X23
```

```
  lmiterm([16 5 5 Q2],1,1);                        % LMI #16: Q2
  lmiterm([16 5 5 Q1],1,-1);                       % LMI #16: -Q1
  lmiterm([16 5 5 0],d*h*X22-X23-X23'+h*Y11-d*h*Y11+Y13+Y13');     % LMI
#16: d*h*X22-X23-X23'+h*Y11-d*h*Y11+Y13+Y13'
  lmiterm([16 6 5 0],h'*Y12'-d'*h'*Y12'-Y13'+Y23);     % LMI #16: h'*Y12'-
d'*h'*Y12'-Y13'+Y23
  lmiterm([16 6 6 Q2],1,-1);                       % LMI #16: -Q2
  lmiterm([16 6 6 0],h*Y22-d*h*Y22-Y23-Y23');      % LMI #16: h*Y22-d*h*Y22-
Y23-Y23'
  lmiterm([16 7 7 0],-R4+t*d*R4);                  % LMI #16: -R4+t*d*R4
  lmiterm([16 8 1 -R1],A*d'*h',1);                 % LMI #16: A*d'*h'*R1'
  lmiterm([16 8 1 -R2],A*h',1);                    % LMI #16: A*h'*R2'
  lmiterm([16 8 1 -R2],A*d'*h',-1);                % LMI #16: -A*d'*h'*R2'
  lmiterm([16 8 1 -R3],A*a'*h',1);                 % LMI #16: A*a'*h'*R3'
  lmiterm([16 8 1 0],A*R4');                       % LMI #16: A*R4'
  lmiterm([16 8 2 -R1],B*d'*h',1);                 % LMI #16: B*d'*h'*R1'
  lmiterm([16 8 2 -R2],B*h',1);                    % LMI #16: B*h'*R2'
  lmiterm([16 8 2 -R2],B*d'*h',-1);                % LMI #16: -B*d'*h'*R2'
  lmiterm([16 8 2 -R3],B*a'*h',1);                 % LMI #16: B*a'*h'*R3'
  lmiterm([16 8 2 0],B*R4');                       % LMI #16: B*R4'
  lmiterm([16 8 3 -R1],F*d'*h',1);                 % LMI #16: F*d'*h'*R1'
  lmiterm([16 8 3 -R2],F*h',1);                    % LMI #16: F*h'*R2'
  lmiterm([16 8 3 -R2],F*d'*h',-1);                % LMI #16: -F*d'*h'*R2'
  lmiterm([16 8 3 -R3],F*a'*h',1);                 % LMI #16: F*a'*h'*R3'
  lmiterm([16 8 3 0],F*R4');                       % LMI #16: F*R4'
  lmiterm([16 8 4 -R1],G*d'*h',1);                 % LMI #16: G*d'*h'*R1'
  lmiterm([16 8 4 -R2],G*h',1);                    % LMI #16: G*h'*R2'
  lmiterm([16 8 4 -R2],G*d'*h',-1);                % LMI #16: -G*d'*h'*R2'
  lmiterm([16 8 4 -R3],G*a'*h',1);                 % LMI #16: G*a'*h'*R3'
  lmiterm([16 8 4 0],G*R4');                       % LMI #16: G*R4'
  lmiterm([16 8 8 R1],.5*d*h,-1,'s');              % LMI #16: -d*h*R1 (NON
SYMMETRIC?)
  lmiterm([16 8 8 R2],.5*h,-1,'s');                % LMI #16: -h*R2 (NON
SYMMETRIC?)
  lmiterm([16 8 8 R2],.5*d*h,1,'s');               % LMI #16: d*h*R2 (NON
SYMMETRIC?)
  lmiterm([16 8 8 R3],.5*a*h,-1,'s');              % LMI #16: -a*h*R3 (NON
SYMMETRIC?)
  lmiterm([16 8 8 0],-R4);                         % LMI #16: -R4

  Math=getlmis;

    [tmin,xfeas]=feasp(Math)
     P=dec2mat(Math,xfeas,P)
     Q1=dec2mat(Math,xfeas,Q1)
     Q2=dec2mat(Math,xfeas,Q2)
     Q3=dec2mat(Math,xfeas,Q3)
     R1=dec2mat(Math,xfeas,R1)
     R2=dec2mat(Math,xfeas,R2)
     R3=dec2mat(Math,xfeas,R3)
```

BIOGRAPHY

BIOGRAPHY

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